A Novel Hybrid Algorithm of Black Hole and Differential Evolution for High Dimensional Electromagnetic Optimal Problems

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In order to solve high dimensional problems simply and efficiently, a novel hybrid algorithm of differential evolution algorithm and black hole algorithm has been proposed. The differential evolution algorithm owns faster convergence, while the black hole based optimal design algorithm dominates its counterpart in diversity and flexibility. Therefore, due to the advantages combination of these two algorithms, the proposed hybrid algorithm owns better convergence and robustness than either of the two algorithms on solving high dimensional problems. The corresponding optimal performance is deeply investigated through numerical experiments on benchmark test functions and engineering applications.

Index Terms—**Black hole phenomenon, differential evolution, high-dimensional optimization problem, electromagnetic device**

I. Introduction

OPTIMIZATION ALGORITHMS has been widely used in various disciplines. Among the current optimization algorithms, meta-heuristic algorithms such as genetic algorithm and particle swarm optimization become more and more popular by their flexibility and utility. They have been proved to be effective in dealing with the common simple engineering problems with low dimension [1]. However, as the dimension number increased, the aforementioned algorithms are easy to fall into local optimal values, and may have premature or result in low efficiency. Furthermore, for the practical engineering problems, the relationship of objective performance and design variables is implicit. The design problems may be complex, strongly nonlinear, and multimodal.

Recently, a new population-based method, the black hole based optimization (BHBO) algorithm, has been proposed [2], which is parameter-free and only needs mathematic equations in star update and sucking condition. It is very easy to use and timesaving. The BHBO has been proved to be suitable for solving low dimensional electromagnetic problem [2]. Until now, there is no relevant research of BHBO algorithm dealing with high dimensional problems.

Among existing evolutionary algorithms, the differential evolution (DE) has been proved to be simple and faster with fewer control parameters [3], it can always get high accuracy solution when dealing with low dimensional problems. However, the performance of solving high dimensional problems (more than 50) hasn't been investigated [3]-[4]. Considering this situation, it is urgent to develop a simple and effective algorithm.

The main contribution of this paper is to propose and apply a new meta-heuristic technique based on black hole and differential evolution for seeking global optimum of complex and high dimensional electromagnetic problems effectively, flexibly and stability. Combing two algorithms with simple structure, few control parameters, and faster convergence, the proposed hybrid algorithm is predicted to own better performance than either of the single one.

II. The Proposed Hybrid Algorithm

A. Black hole based optimization algorithm

In space, the star or other objects within the Schwarzschild radius [2] will be absorbed by the black hole. And the BHBO algorithm is proposed based on this phenomenon.

Firstly, randomly generate N_p individuals called stars. The star with the best objective value is selected as black hole $\mathbf{x}_{\mathrm{BH}}$. Then all stars move towards black hole as:

$$\mathbf{x}_i^{g+1} = \mathbf{x}_i^g + rand(0,1)\times(\mathbf{x}_{BH}^g - \mathbf{x}_i^g), \quad \mathbf{x}_i^g \neq \mathbf{x}_{BH}^g \tag{1}$$

where $\mathbf{x}_i^g \in R^n$ is the ith star at the gth generation. After movement, a star reaching a position with better objective value than $\mathbf{x}_{BH}^g$ will be the new black hole $\mathbf{x}_{BH}^{g+1}$. One star will die if the distance between it and the black hole is less than the Schwarzschild radius (R) calculated as:

$$R = \left| f\left(\mathbf{x}_{BH}^g\right)\right| \Big/ \sum_{i=1}^{N_p} \left| f\left(\mathbf{x}_i^g\right)\right| \tag{2}$$

where $f(\ )$ is the objective function to be optimized. To keep the number of stars constant, a new star is generated in design space once a star dies.

B. Differential evolution algorithm

The DE algorithm includes initial, mutation, crossover and selection [5]. Calculation process of the algorithm shows as follows, while the number of vectors N_p is a constant.

Firstly, N_p target vectors with n-dimensional parameter $\mathbf{x}_i^g(x_{i,1}^g, x_{i,2}^g, \ldots, x_{i,n}^g)$ where i=1,2,…, N_p are generated randomly in the whole design space. Then, the mutational vectors $\mathbf{v}_i^{g+1}$ are generated as follows:

$$\mathbf{v}_i^{g+1} = \mathbf{x}_{r1}^g + \mathrm{F}\times(\mathbf{x}_{r2}^g - \mathbf{x}_{r3}^g), \ \ r1 \neq r2 \neq r3 \neq i \tag{3}$$

where F is a weight coefficient.

After that, the trial vectors $\mathbf{u}_i^{g+1}$ are generated as follows:

$$u_{i,j}^{g+1} = \begin{cases} v_{i,j}^{g+1} & \text{if}\,(rand_j(0,1) \le C_r \text{ or } j = j_{rand}) \\ x_{i,j}^g & \text{otherwise} \end{cases} \tag{4}$$

where Cr is a crossover constant, $rand_j(\ )$ is a random number on the jth dimension, and j_{rand} is a randomly dimension index.

At last, the better objective values of trial vector $\mathbf{u}_i^{g+1}$ or target vector $\mathbf{x}_i^g$ will go to next generation.

C. The proposed novel hybrid algorithm

The proposed novel hybrid black hole and differential evolution optimization (BH-DEO) algorithm has combined advantages of two algorithms to solve high dimensional, nonlinear and complex problems. For one optimization problem of minimizing objective function subject to some constraints, the detailed BH-DEO algorithm is explained as:

Step1: Initialization. Randomly generate N_p target vectors $\mathbf{x}_i^g$ (i=1, 2, …, N_p). The target vector with the best fitness value is chose as black hole $\mathbf{x}_{BH}^0$. Set the maximum generation number as $g_{\max}$.

Step2: Mutation and Crossover. Same as in the DE, the mutation and crossover are implemented by (3) and (4).

Step 3: Movement. The target vectors move as in (1) to generate another group of trial vectors $\mathbf{b}_i^{g+1}$.

$$\mathbf{b}_i^{g+1} = \mathbf{x}_i^g + rand(0,1) \times (\mathbf{x}_{BH}^g - \mathbf{x}_i^g) \tag{5}$$

Step 4: Survival criterion. In the current generation, the ith trial vectors $\mathbf{u}_i^{g+1}$ or $\mathbf{b}_i^{g+1}$ which gives better objective value will be selected as target vectors $\mathbf{x}_i^{g+1}$ for next generation. Then update the black hole $\mathbf{x}_{BH}^{g+1}$.

- If both of them are feasible, select survivor as:

$$\mathbf{x}_i^{g+1} = \begin{cases} \mathbf{u}_i^{g+1} & f(\mathbf{u}_i^{g+1}) \le f(\mathbf{b}_i^{g+1}) \\ \mathbf{b}_i^{g+1} & f(\mathbf{u}_i^{g+1}) > f(\mathbf{b}_i^{g+1}) \end{cases} \tag{6}$$

- If one of them is feasible, select the feasible one;
- If both of them are infeasible, select that with lower summation of constraint violations.

Step 5: Vector correction. If (7) is satisfied or target vector beyonds the design space, the corresponding target vector will be absorbed and a new target vector will be generated to keep N_p target vectors.

$$\left\| \mathbf{x}_i^{g+1} - \mathbf{x}_{BH}^{g+1} \right\| < R \tag{7}$$

Step 6: Termination. If the maximum generation $g_{\max}$ is reached, the program stop, otherwise, go to Step 2.

III. Numerical Optimization

Four high-dimensional and multimodal functions in Table I are selected to investigate performance of BH-DEO algorithm. Each test function has been tested for 20 times.

Table I High-dimensional test functions

Functions [a]	Range	Optimum
Acklcy : $y = -20\exp[-0.2\sqrt{1/n\sum_{i=1}^{n} x_i^2}] - \exp[\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)] + 20 + e$	[-32,32]	0
Griewank : $y = \sum_{i=1}^{n} x_i^2 / 4000 - \prod_{i=1}^{n}[\cos(x_i/\sqrt{i}) + 1]$	[-600,600]	0
Rastrigin : $y = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)]$	[-5.12,5.12]	0
Schwelfel : $y = \sum_{i=1}^{n}\lvert x_i\rvert + \prod_{i=1}^{n}\lvert x_i\rvert$	[-10,10]	0

a - For dimensions 30, 60, and 100, *Ns* is set three times of the dimension number. Meanwhile, the g_{max} is set as 2000, 3000 and 4000 respectively.

From Table II, as the dimension increases, it can be seen that the optimization ability of BHBO and DE algorithms have been decreased. However, the proposed BH-DEO algorithm can generally maintain a stable ability. The BHBO failed to find the optimal solution of all the test functions except the Griewank, and shows low robustness for high dimensional functions. DE fails to optimize the Rastrigin at n=30 while it failed to optimize the Acklcy, Rastrigin, and Schwelfel at n=100. On the contrary, the proposed BH-DEO algorithm can always find better solutions with good accuracy than its counterparts. The robustness and effectiveness of the BH-DEO algorithm have been demonstrated on the complex and high dimensional functions. Further investigations on the engineering applications will be given in the full paper.

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References

[1] M.-T. Pham, Z. Ren, W. Li, and C.-S. Koh, "Optimal design of a Thomson-coil actuator utilizing a mixed-integer-discrete-continuous variables global optimization algorithm," *IEEE Trans. Magn*, 2011, vol. 47, no.10, pp. 4163-4166.

[2] Hotssem R.E.H. Bouchekara, "Optimal design of electromagnetic devices using a black-hole-based optimization technique," *IEEE Trans. Magn*, 2013, vol. 49, no.12, pp. 5709-5714.

[3] G. C. Tenaglia, and L. Lebensztajn, "A multiobjective approach of differential evolution optimization applied to electromagnetic problems," *IEEE Trans. Magn*, 2014, vol. 50, no.2, Article No. 7015404.

[4] N. Baatar, D. Zhang, and C.-S. Koh, "An improved differential evolution algorithm adopting λ-best mutation strategy for global optimization of electromagnetic devices," *IEEE Trans. Magn*, 2014, vol. 50, no.2, Article No. 7017504.

[5] R.Storn, and K. Price, "Differential evolution – a simple and efficient heuristic for global optimization over continuous space," *Journal of Global Optimization*, 1997, pp. 341-359.

Table II Result comparsion of the different algorithms

Funs	Dim. n	30			60			100		
	Method	BHBO	DE	BH-DEO	BHBO	DE	BH-DEO	BHBO	DE	BH-DEO
Acklcy	Worst	5.2560	7.14E-11	**8.05E-09**	5.7485	9.45E-06	**1.69E-09**	8.6163	0.0020	**5.67E-09**
	Mean	3.7286	2.77E-11	**2.12E-09**	4.4067	5.21E-06	**7.63E-10**	5.0603	0.0014	**2.97E-09**
	Best	2.4959	6.44E-12	**1.55E-10**	3.0604	3.46E-06	**2.06E-10**	3.4042	0.0008	**1.21E-09**
Griewank	Worst	1.90E-12	1.22E-22	**1.36E-15**	3.64E-11	2.11E-11	**3.39E-13**	2.56E-10	1.51E-06	**4.17E-11**
	Mean	2.97E-13	4.79E-23	**2.72E-16**	9.67E-12	6.68E-12	**5.59E-14**	1.09E-10	7.89E-07	**1.45E-11**
	Best	2.29E-15	7.03E-24	**1.54E-17**	5.57E-13	1.87E-12	**6.83E-15**	2.15E-11	4.41E-07	**2.08E-12**
Rastrigin	Worst	125.9228	184.9400	**8.48E-07**	294.8560	465.7691	**1.02E-07**	623.8676	863.3745	**1.72E-08**
	Mean	62.0315	155.9874	**4.55E-08**	179.3551	442.0467	**7.95E-09**	340.4088	832.1756	**9.30E-10**
	Best	26.8965	127.6875	**7.78E-13**	34.9648	407.4440	**3.48E-13**	144.4690	763.0413	**1.85E-12**
Schwelfel	Worst	5.7649	7.82E-10	**3.59E-10**	192.0266	1.81E-04	**5.07E-11**	17.1612	0.0511	**2.40E-11**
	Mean	2.4991	2.90E-10	**9.34E-11**	14.6992	9.73E-05	**1.90E-11**	10.2001	0.0286	**9.01E-12**
	Best	0.7163	7.05E-11	**1.44E-11**	2.1927	5.40E-05	**2.49E-12**	5.8596	0.0179	**1.84E-12**